Holographic quantum simulation of entanglement renormalization circuits

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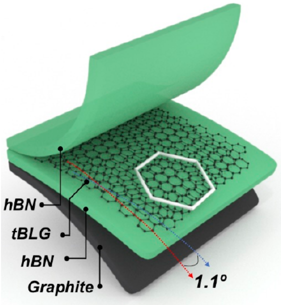
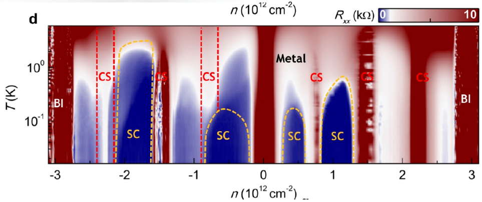
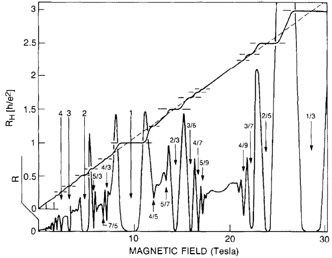
Motivation: condensed matter physics

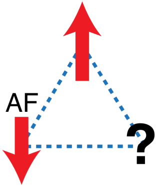




Goal: solve

$$H\left|\psi_0\right\rangle = E_0\left|\psi_0\right\rangle$$

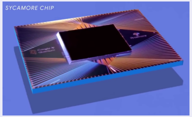


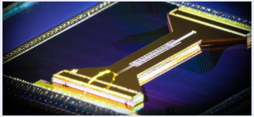


"Nature is quantum, goddamn it!
So if we want to simulate it,
we need a quantum computer."

R. Feynman, 1981

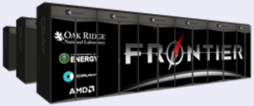
NISQ era devices





$\leftrightarrow$

Exa-Scale Supercomputers



Variational Quantum Eigensolver (VQE)

Goal

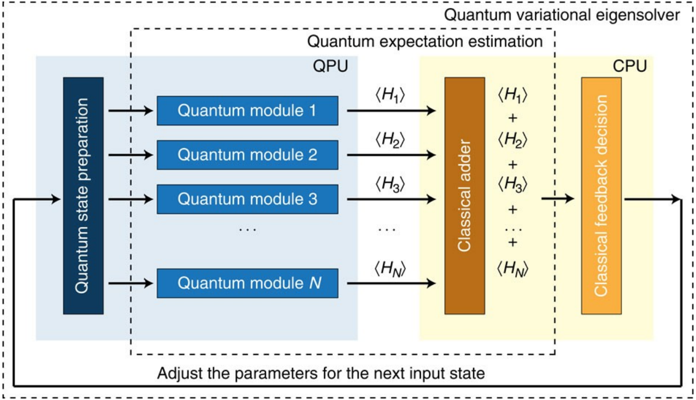
find ground state

$$H\,|\psi_0\rangle = E_0\,|\psi_0\rangle$$

of a *physical* Hamiltonian

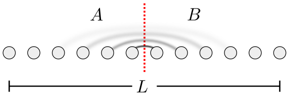
$$H = \sum_i H_i$$

⇒ known good ansatz: tensor networks



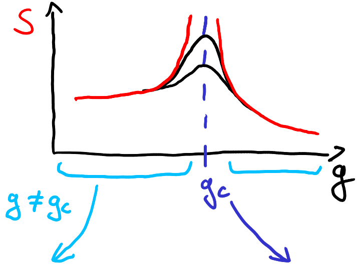
Peruzzo *et al.*, Nature Comm. **5** 4213 (2014)

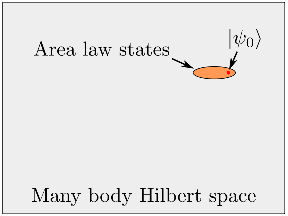
Justification of tensor networks: entanglement entropy



$$S = -\operatorname{Tr} \varrho_A \log(\varrho_A)$$

$$\varrho_A = \operatorname{Tr}_B |\psi\rangle\langle\psi|$$





Area law

for ground states of
gapped, *local* Hamiltonians

$$S(L) \propto \text{area of cut} \propto L^{d-1}$$

Hastings J.Stat.Mech. 2007

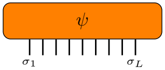
critical (scale-invariant) points

conformal field theory predicts in 1D

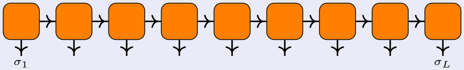
$$S(L) = \frac{c}{6} \log(L) + \text{const.}$$

Calabrese, Cardy J.Stat.Mech. 2004

Tensor Network Ansätze in 1D

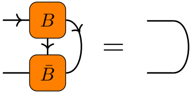
$|\psi\rangle = \sum_{\sigma_1,\dots,\sigma_L}$  $|\sigma_1,\dots,\sigma_L\rangle$

Matrix product states (MPS)

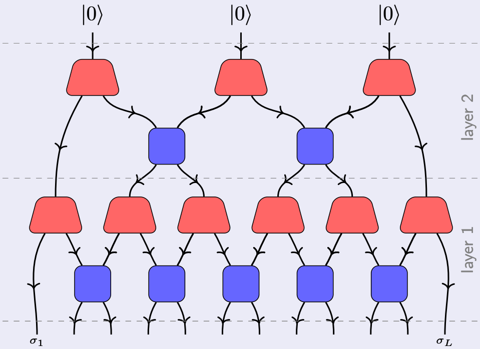


with bounded entropy $S \leq \log \chi$

orthogonality conditions:

$\sum_\sigma B_\sigma B_\sigma^\dagger =$  $= \mathbb{1}$

Multiscale Entanglement Renormalization Ansatz (MERA)



can capture $S \propto \log(L)$

Trapped-Ion QCCD @ Honeywell/Quantinuum

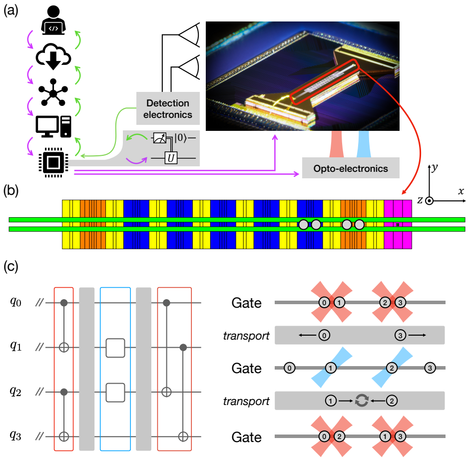
Pino *et al.*, Nature **592** 209 (2021)

- ^{171}Yb$^+$ qubit - ^{138}Ba$^+$ coolant
- transport qubits for interactions between gate/auxiliary zones

\+ low error rates per gate
  - single-qubit: 1.1×10^{-4}
  - two-qubit: 8.0×10^{-3} (in spring 2021)

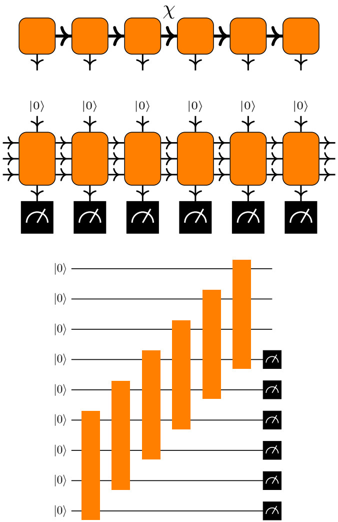
\+ can measure/reset qubit mid-circuit

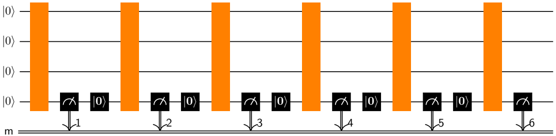
− low number of qubits $N \leq 10$



⇒ ideal for holographic simulation

Recap: Holographic MPS state preparation





⇒ "holographic": reset and reuse qubit

Kim arXiv:1702.02093

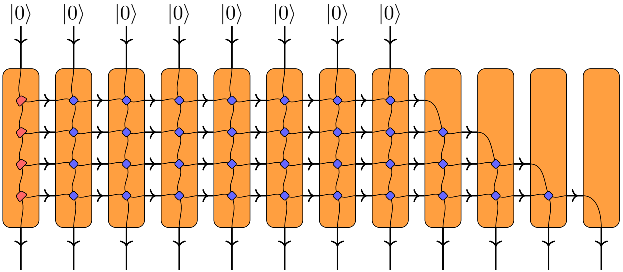
- requirements

$$N_{\text{qubits}} = 1 + \log_2(\chi) \propto \text{const. in } L$$
$$N_{\text{gates}} \propto L$$

Foss-Feig *et al.*, PRR **3**, 033002 (2021)

Decomposition of general $U(\chi)$

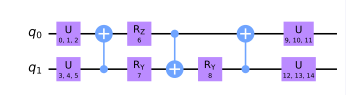
- general $U(\chi)$ has prohibiting $\chi^2 = 4^{N_\text{qubits}}$ real parameters
- instead decompose with sequence of 2-qubit gates, e.g. brick-wall or ladder fashion



general 2-qubit gate

Vatan, William, PRA (2004)

15 real parameters

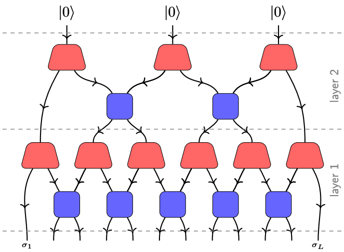


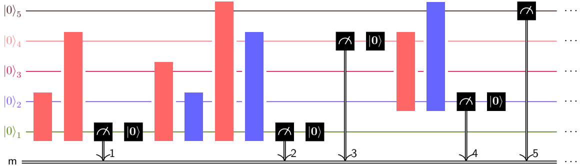
$\Rightarrow$ composition: 6 param.

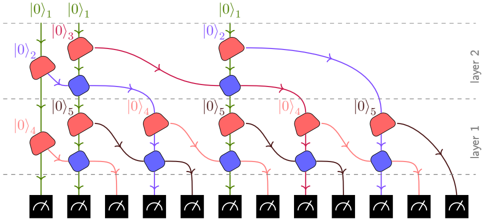
Lin *et al.*, PRX Quantum **2**, 010342 (2021)

Haghshenas *et al.*, PRX **12**, 011047 (2021)

Holographic MERA state preparation







- requirements

$$N_{\text{qubits}} = 1 + 2 \cdot N_{\text{layers}}$$

$$N_{\text{gates}} = \sum_{i=1}^{N_{\text{layers}}} \left(\frac{L}{2^{i-1}} - 1 \right) < 2L$$

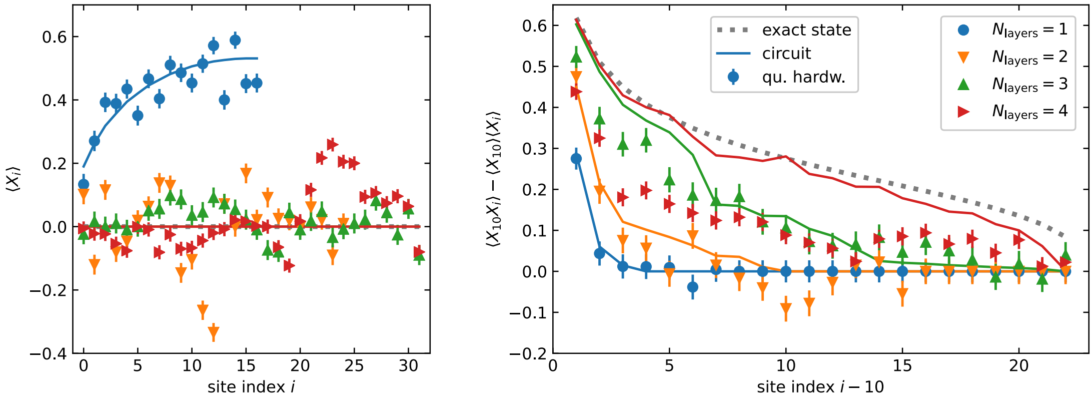
- here: restrict to $\chi = 2$ MERA
  $\Rightarrow$ only two-qubit unitaries

Our Setup

1. classically optimize tensor network for given H
   - find high-precision reference MPS with DMRG, then optimize MERA to have maximal overlap.
   - could be optimized directly on quantum hardware with VQE
2. convert to circuit
   - extend V and parametrize U
3. run circuit on quantum hardware
   - circuit compilation at Honeywell
4. evaluate expectation values and correlation functions
   - separate runs for X/Z observables

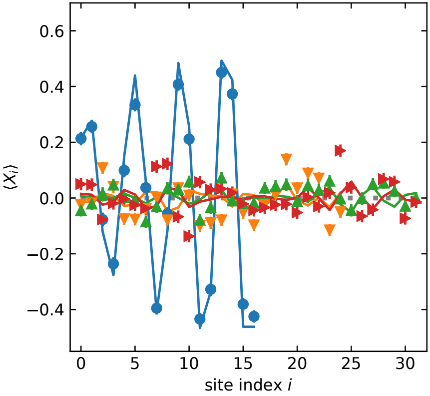
$$H$$
$$\downarrow$$
$$|\psi_0^{\mathrm{exact}}\rangle$$
$$\approx\downarrow$$
$$|\psi_0^{\mathrm{MERA}}\rangle$$
$$\downarrow$$
circuit
$$\approx\downarrow$$
measurements

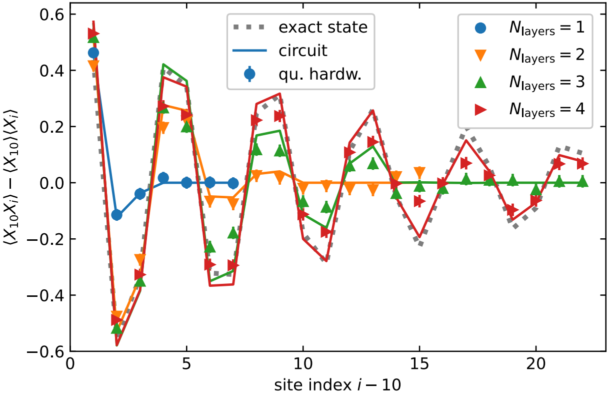
Transverse field Ising model $H = \sum_i -(X_i X_{i+1} + Z_i)$



⇒ systematic errors beyond measurement noise

Self-dual TFI $H = \sum_i -(X_i X_{i+1} + Z_i) + 4(Z_i Z_{i+1} + X_i X_{i+2})$



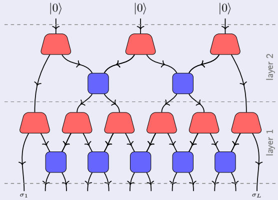


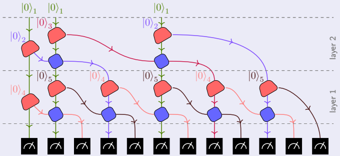
$\Rightarrow$ observe correlations beyond N_{qubits}

finite-layer MERA causes correlation cutoff - can we do better?

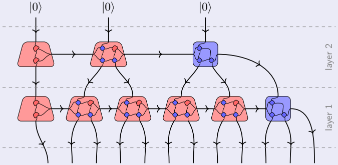
Between MERA and MPS: generalized MERA (gMERA)

MERA

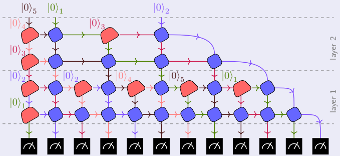




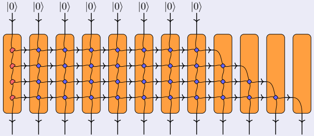
gMERA

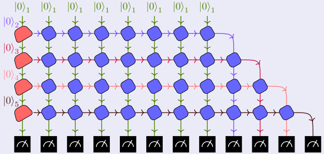


Bal *et al.*, Phys. Rev. B **94**, 205122 (2016)

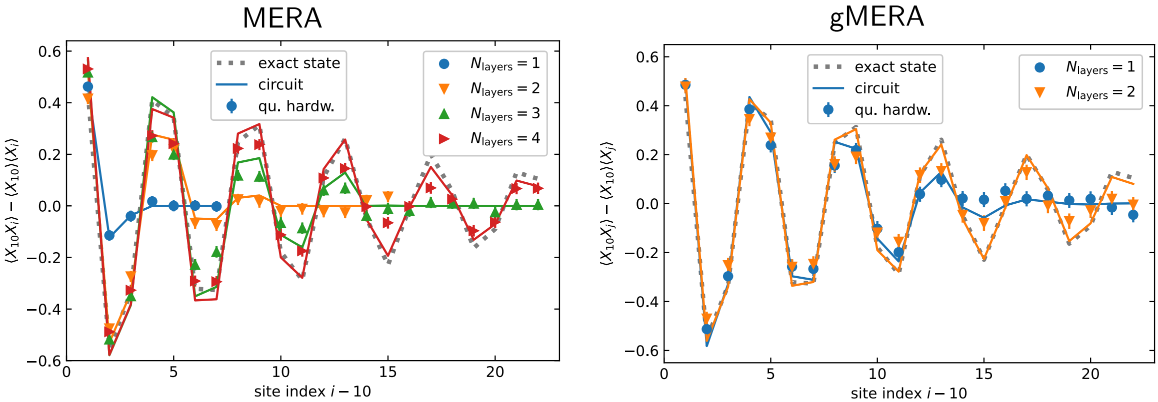


MPS





Self-dual transverse-field Ising model: MERA *vs* gMERA

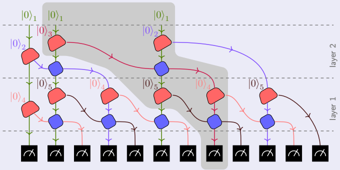


$\Rightarrow$ gMERA is more expressive at low number of layers

Noise considerations

Kim, Swingle, arXiv:1711.07500 ⇒ MERA is noise resilient

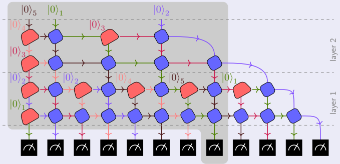
MERA



naive $\mathcal{O}(\epsilon \cdot N_{\text{layers}} \cdot 3)$

$\mathcal{O}(\epsilon)$

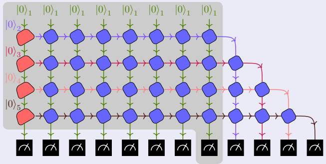
gMERA



naive $\mathcal{O}(\epsilon \cdot 2 \cdot L)$

$\mathcal{O}(\epsilon \cdot 2 \cdot \xi)$

MPS



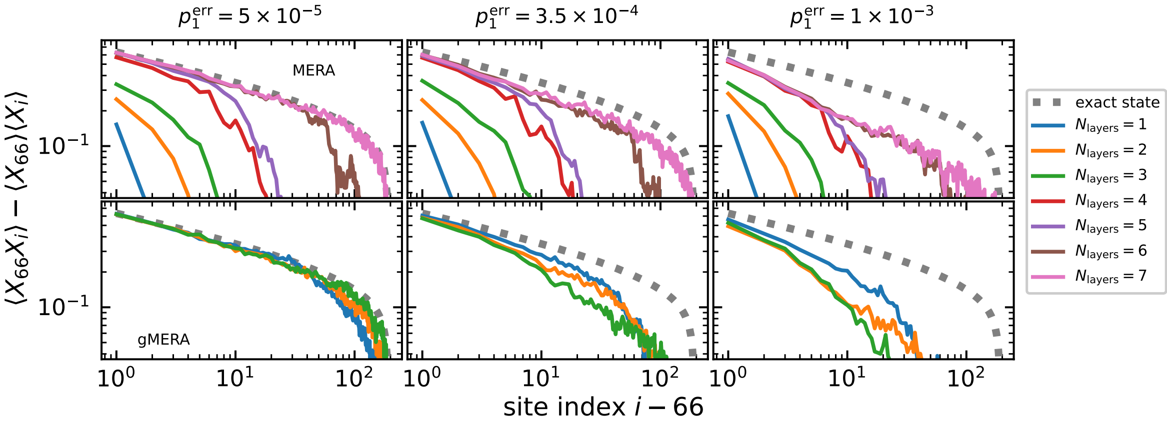
naive $\mathcal{O}(\epsilon \cdot N_{\text{layers}} \cdot L)$

$\mathcal{O}(\epsilon \cdot N_{\text{layers}} \cdot \xi)$

see also Sewell, Jordan, arXiv:2109.09787

Noise simulations

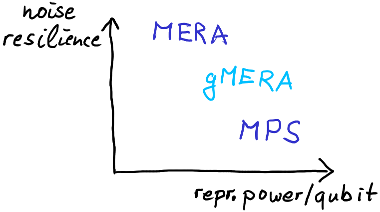
depolarizing noise with 2-qubit error rate $p_2^{\mathrm{err}} = 10 p_1^{\mathrm{err}}$



$\Rightarrow$ *best* network depends on available qubits and noise levels

- gMERA has more representational power, but MERA is more noise resilient

Summary



- taylored to (Honeywell's) trapped ions:
  small number of qubits,
  ability to reset, and low noise per gate
- propose gMERA between MERA and MPS
- optimal network choice depends on available qubit number and noise

Outlook

- efficiency of optimization
- time evolution
- isoTNS in 2D

Thank you
for your attention!

arXiv:2203.00886

Appendix

Noise simulations MPS vs gMERA